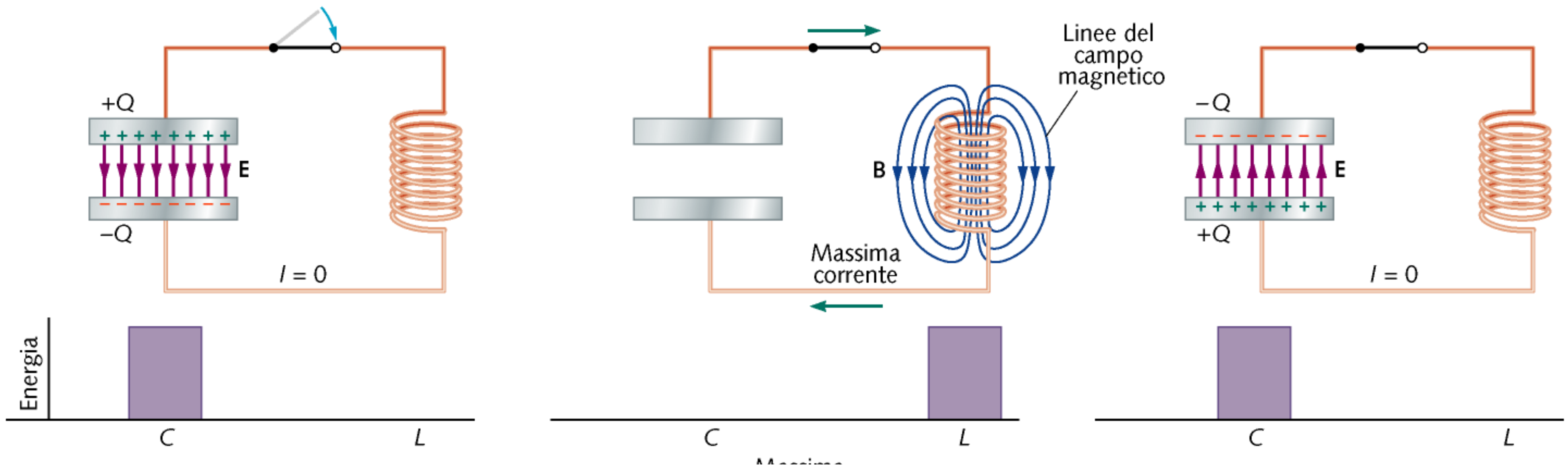
Circuito LC

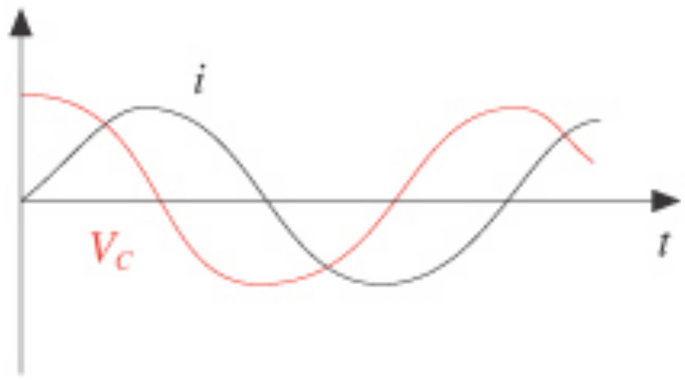


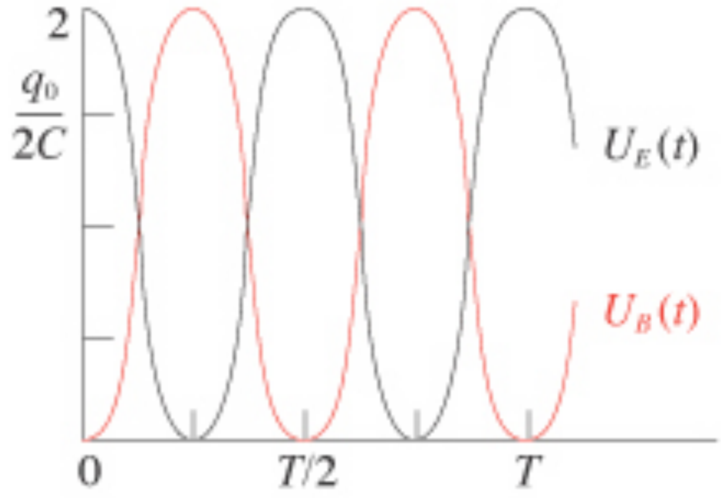
$$L\frac{dI}{dt}+\frac{q}{C}=0$$

$$L\frac{d^2q}{dt^2}+\frac{q}{C}=0$$

$$q(t)=Q\cos(\omega t+\phi)$$

$$\omega=\frac{1}{\sqrt{LC}} \qquad \frac{1}{\sqrt{[H][C]}}=\sqrt{\frac{[A][V]}{[s][V][Q]}}=[s^{-1}]$$





$$V(t) = \frac{q}{C} = \frac{Q}{C}\cos(\omega t + \phi)$$

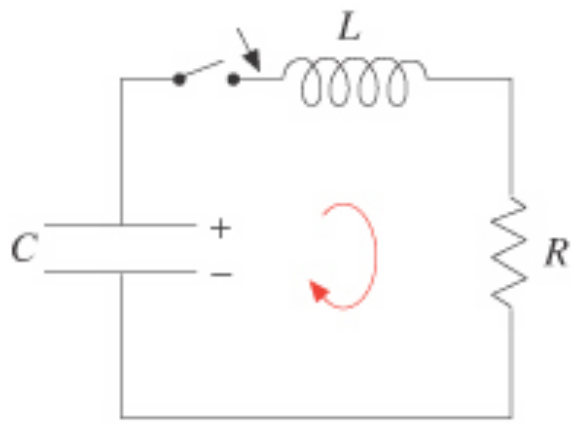
$$I(t) = \frac{d\ q}{dt} = -Q\omega \sin(\omega t + \phi)$$

$$U_E(t) = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi)$$

$$U_B(t) = \frac{LI^2}{2} = \frac{LQ^2\omega^2}{2}\sin^2(\omega t + \phi) = \frac{LQ^2}{2LC}\sin^2(\omega t + \phi)$$

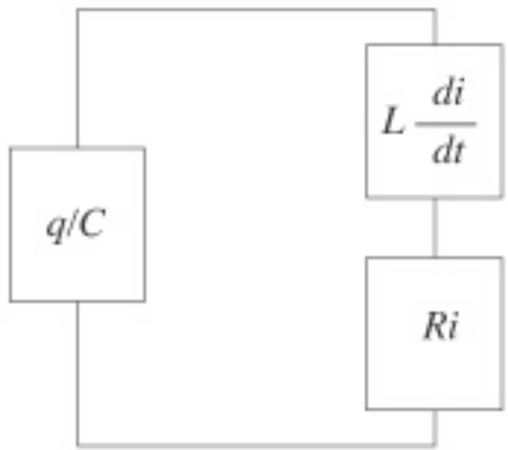
$$U_E(t) + U_B(t) = \frac{Q^2}{2C}$$

Circuito RLC



$$L\frac{dI}{dt}+\frac{q}{C}+RI=0$$

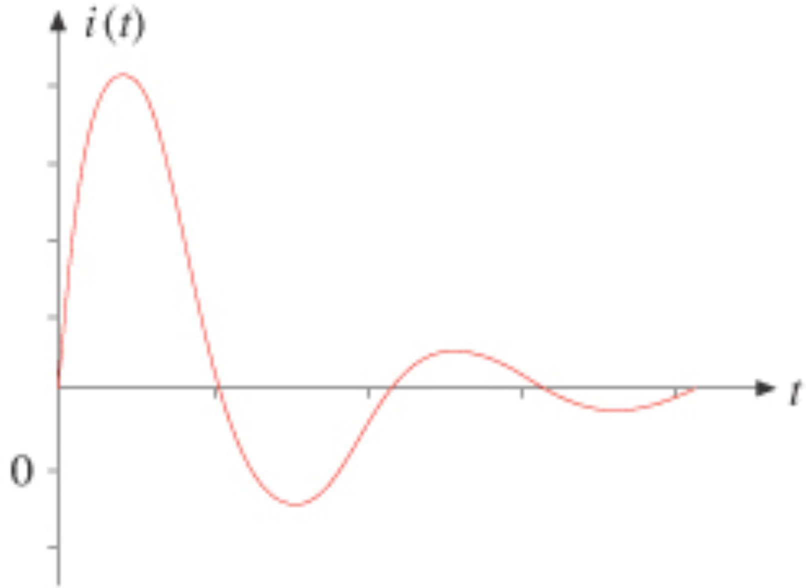
$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=0$$



$$q(t)=Qe^{-\frac{R}{2L}t}\cos(\omega' t+\phi)$$

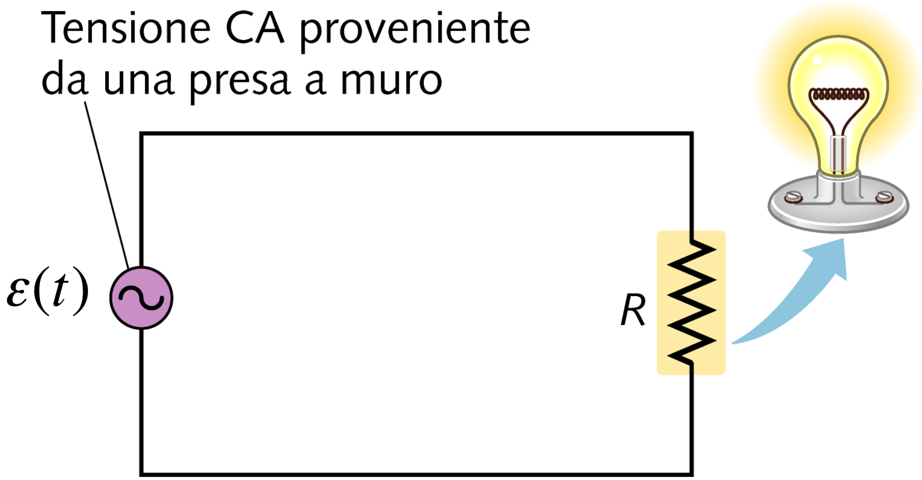
$$\omega'=\sqrt{\omega^2-\left(\frac{R}{2L}\right)^2}$$

Circuito RLC



$$E_C = \frac{q^2}{2C} = \frac{Q^2}{2C} e^{-\frac{R}{L}t} \cos^2(\omega' t + \phi)$$

Carico resistivo con corrente alternata



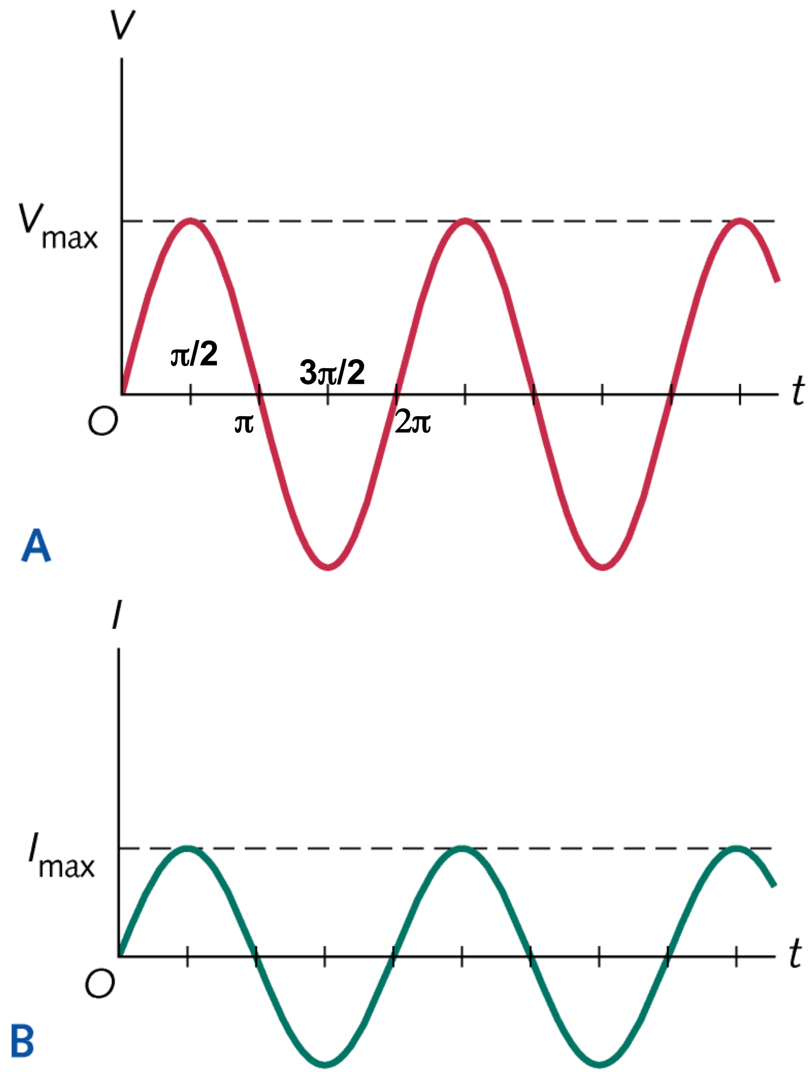
$$\varepsilon(t) = V_{MAX}\sin(\omega t)$$ f.e.m. alternata

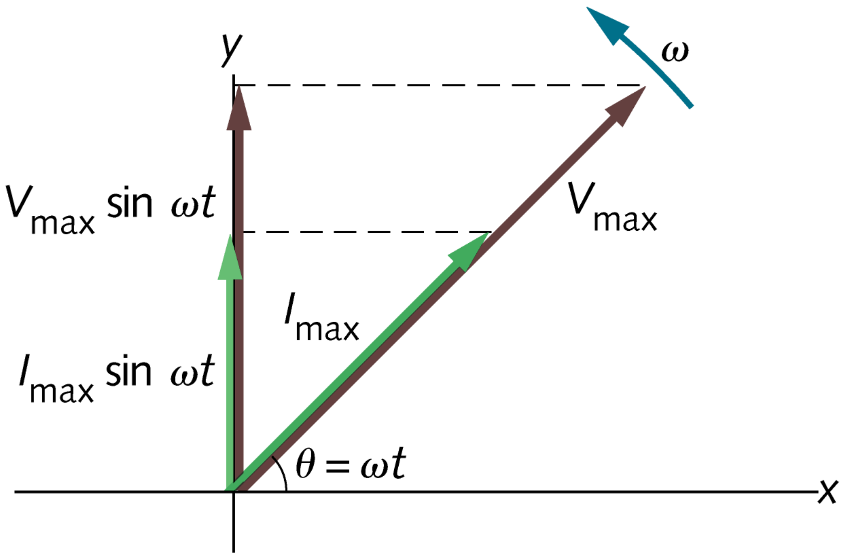
$$V_R = iR = \varepsilon(t)$$

$$V_R(t) = V_{MAX}\sin(\omega t)$$

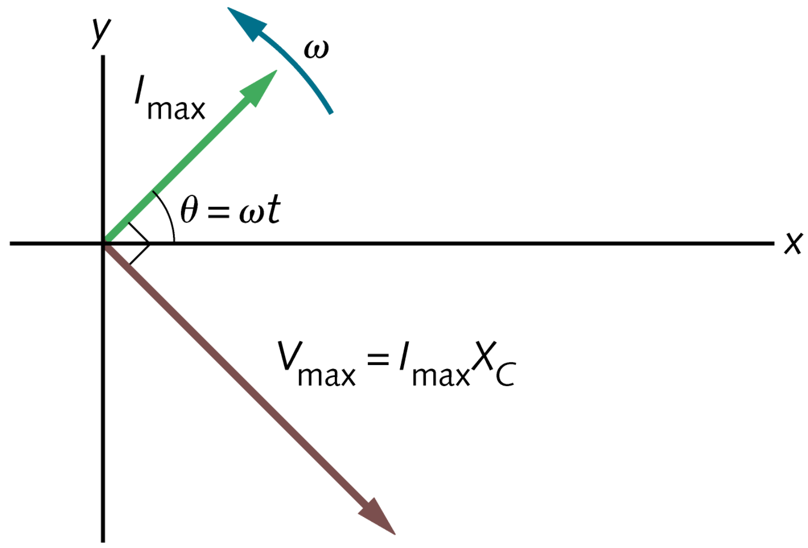
$$I(t) = \frac{V_{MAX}}{R}\sin(\omega t) = I_{MAX}\sin(\omega t)$$

$$I(t) = \frac{V(t)}{R}$$

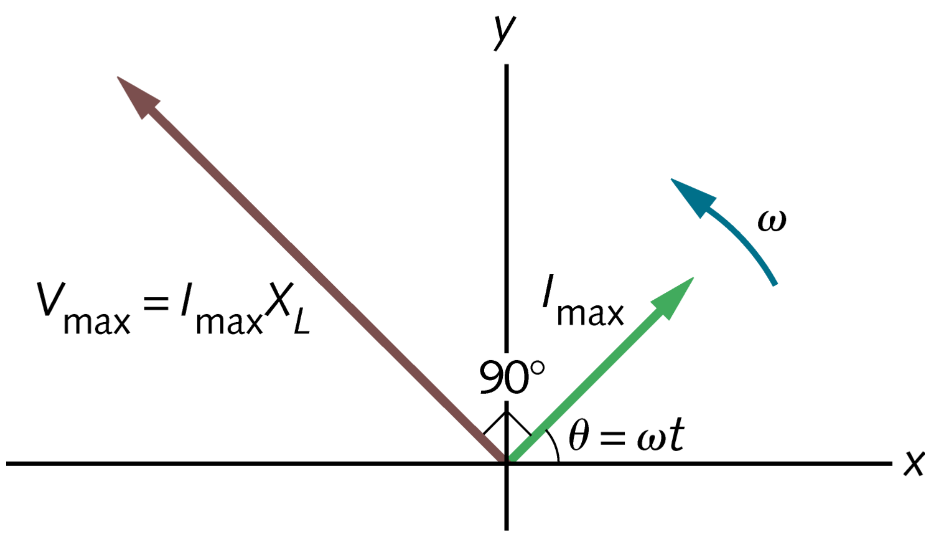




Carico resistivo

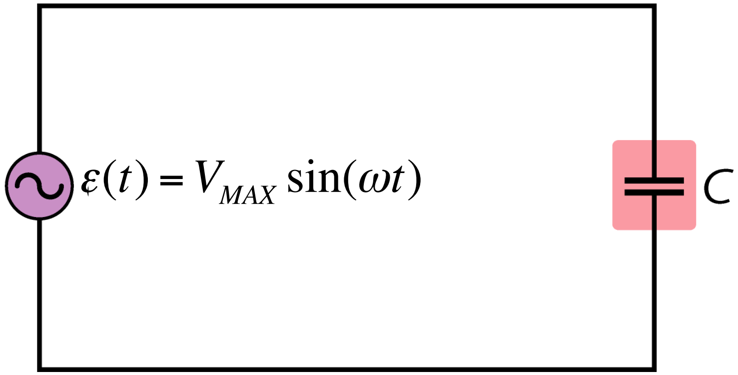


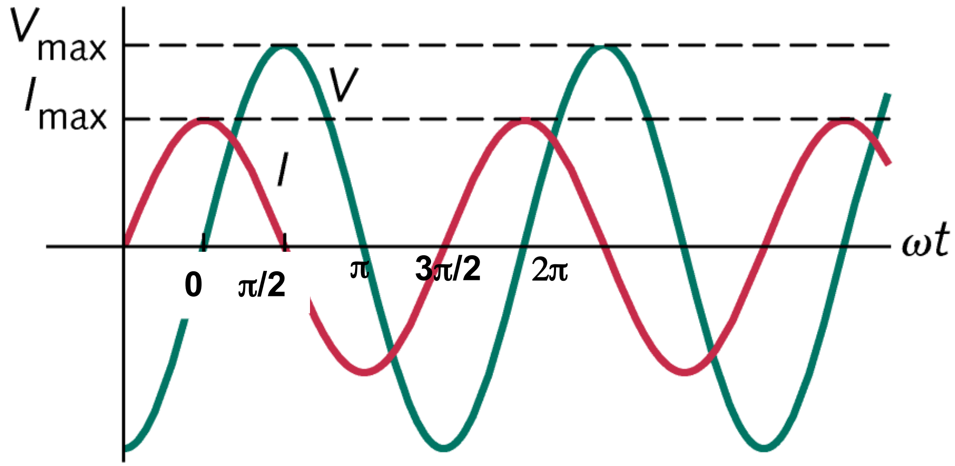
Carico capacitivo



Carico induttivo

Carico capacitivo con corrente alternata





$$V_C = \frac{q}{C} = \varepsilon(t)$$

$$V_C(t) = V_{MAX}\sin(\omega t)$$

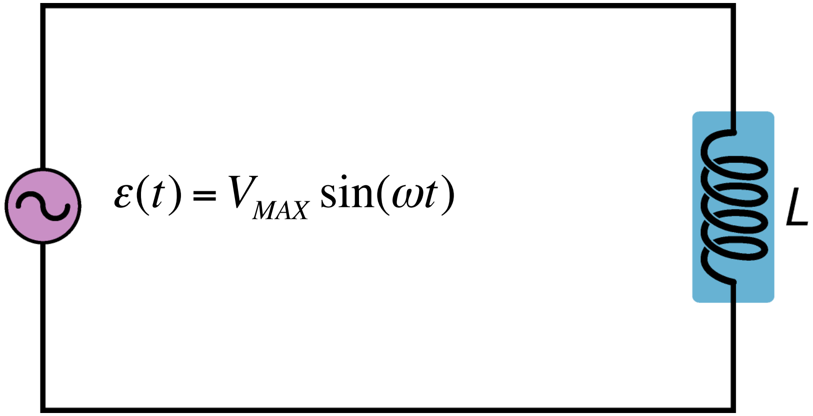
$$q(t) = CV_C(t) = CV_{MAX}\sin(\omega t)$$

$$I(t) = \frac{dq}{dt} = -\omega CV_{MAX}\cos(\omega t) = \omega CV_{MAX}\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$I_{MAX} = \frac{V_{MAX}}{X_C}$$

$X_C = \frac{1}{\omega C}$ Reattanza capacitiva $\qquad \frac{[s]}{[F]} = \frac{[s][V]}{[C]} = \frac{[V]}{[A]} = [\Omega]$

Carico induttivo con corrente alternata



$$V_L = L\frac{dI}{dt} = \varepsilon(t)$$

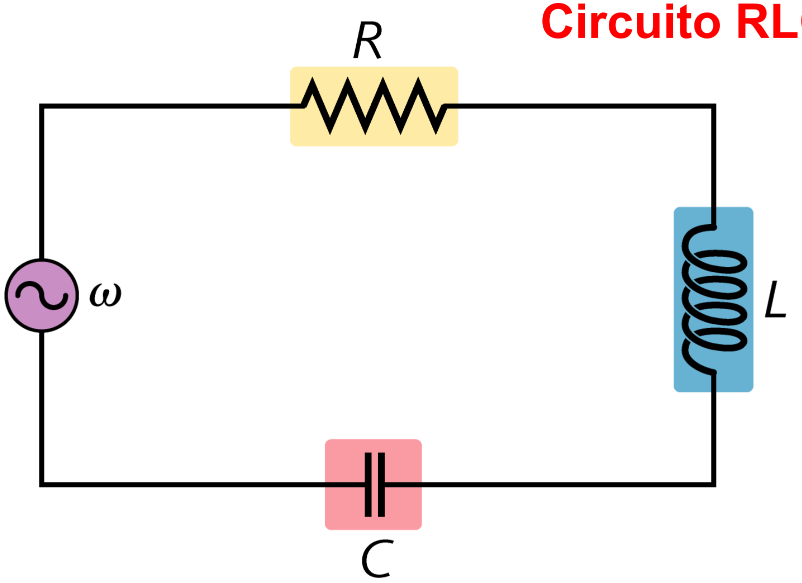
$$V_L(t) = V_{MAX}\sin(\omega t)$$

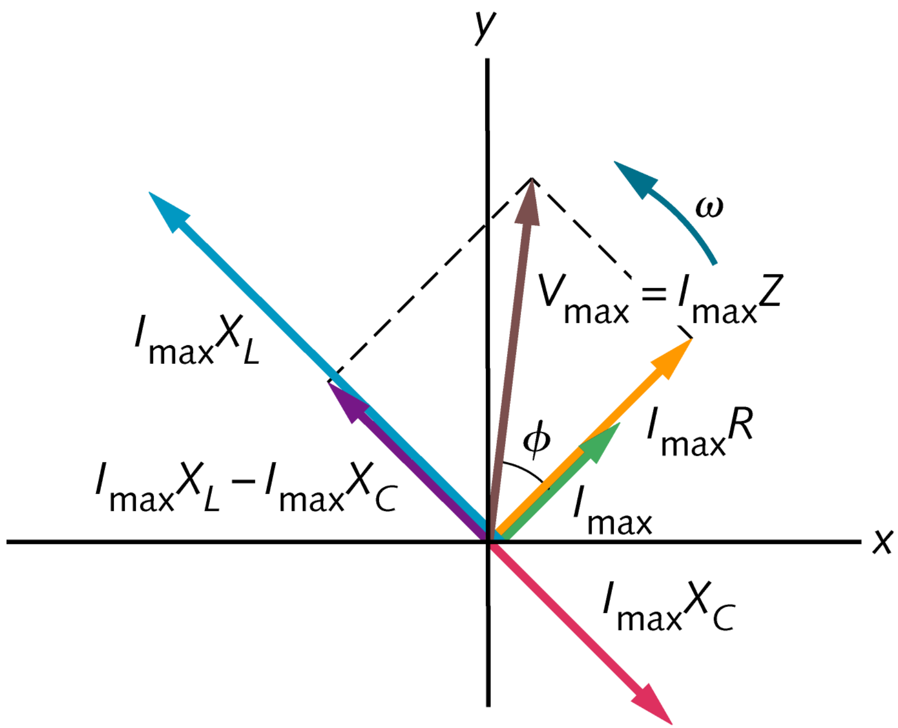
$$I(t) = \int dI(t) = \frac{V_{MAX}}{L}\int \sin(\omega t)dt = -\frac{V_{MAX}}{\omega L}\cos(\omega t) = \frac{V_{MAX}}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$I_{MAX} = \frac{V_{MAX}}{X_L}$$

$X_L = \omega L$ Reattanza induttiva $\dfrac{[H]}{[s]} = \dfrac{[s][V]}{[A][s]} = \dfrac{[V]}{[A]} = [\Omega]$

Circuito RLC in corrente alternata





$$\varepsilon^2 = V_R^2 + (V_L - V_C)^2$$

$$\varepsilon^2 = (IR)^2 + (IX_L - IX_C)^2$$

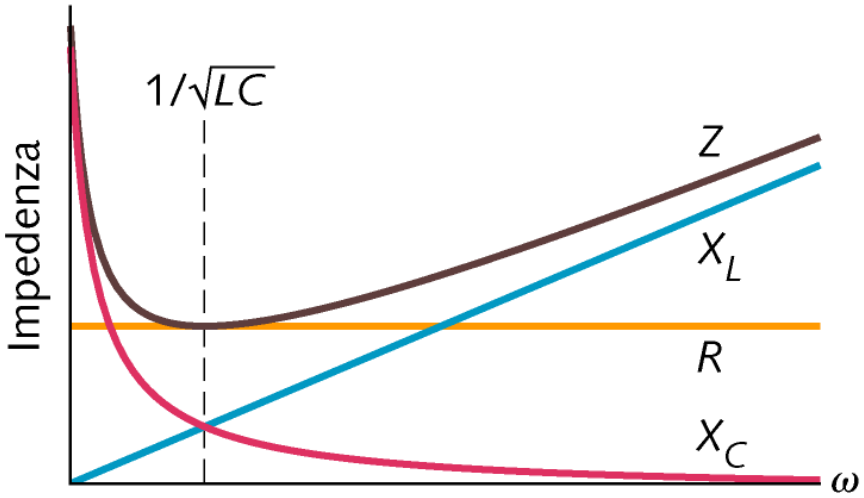
$$I = \frac{\varepsilon}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\varepsilon}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{\varepsilon}{Z}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$ impedenza

$$I = I_{MAX} \sin(\omega t - \phi)$$

$$\tan\phi = \frac{X_L - X_C}{R}$$ fase

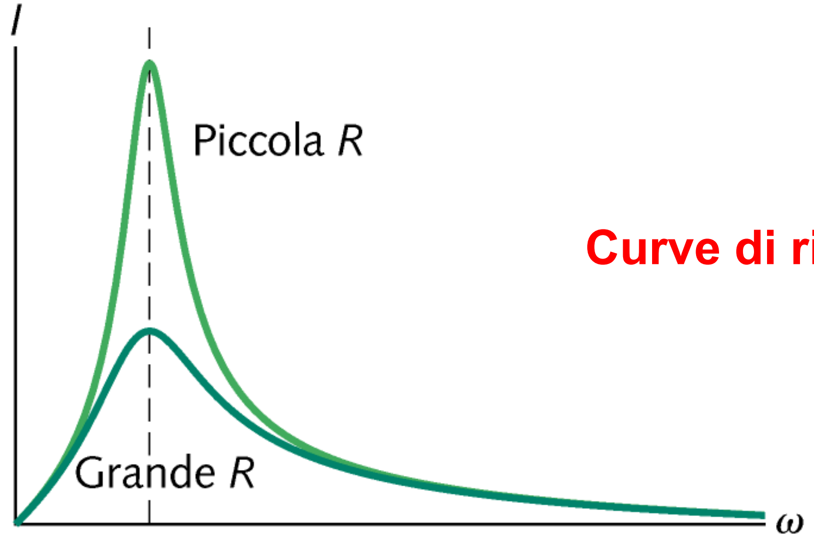
Andamento di Z, X_L, X_C, R in funzione di ω



A

Condizione di risonanza

$$\omega = \frac{1}{\sqrt{LC}}$$



B

Curve di risonanza

Potenza nei circuiti a corrente alternata

$$P(t) = I^2 R = I_{MAX}^2 R \sin^2(\omega t - \phi)$$

$$\overline{P} = \int P(t)\, dt = I_{MAX}^2 R \frac{1}{2\pi} \int_0^{2\pi} \sin^2(\omega t - \phi) dt = \frac{I_{MAX}^2 R}{2}$$

$$I_{eff} = \frac{I_{MAX}}{\sqrt{2}}$$

$$\overline{P} = I_{eff}^2 R = I_{eff}\ I_{eff} R = I_{eff} \frac{V_{eff}}{Z} R = I_{eff} V_{eff} \frac{R}{Z} = I_{eff} V_{eff} \cos\phi$$