Formula for calculating partial cross sections for nuclear reactions of nuclei with $E \gtrsim 200$ MeV/nucleon in hydrogen targets

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In this paper we describe a new formula for calculating the partial cross sections for the production of secondary fragments with energy $\gtrsim 200$ MeV/nucleon in hydrogen targets. This formula and the systematics of these cross sections are based on fragmentation studies using 42 beams of 12 separate nuclei between $Z=6$ and 28. This has resulted in the measurement of more than 100 secondary elemental cross sections and over 300 secondary isotopic cross sections. The systematics of these cross sections allow us to write the cross section formula as a product of three essentially independent terms, one which describes the elemental cross sections, another the isotopic cross sections, and a third term describing the energy dependence. Overall, this formula, which is considerably simpler than earlier semiempirical formulations, is able to predict the cross sections in hydrogen for $Z=4-28$, $A=7-60$ nuclei above ~ 200 MeV/nucleon to an accuracy $\sim 10\%$ or better—a substantial improvement over earlier formulations.

I. INTRODUCTION

The earliest attempts to systematize high-energy cross section measurements into a useful analytical relationship describing the systematics of these reactions are generally credited to Rudstam[1] and Metropolis *et al.*[2] These analytical relationships have been revised and improved by many workers as new cross section data have become available. The direction and detail of this development has generally depended on the particular application. In astrophysics, a comprehensive set of cross sections is essential for interpreting cosmic-ray abundance measurements at earth, in terms of their propagation through the interstellar medium (93% hydrogen) and related inferences about their source composition and the nucleosynthesis and acceleration processes in the source. A systematic description of the cross sections is also necessary for interpreting the abundances of various isotopes in meteorites, or on lunar and planetary surfaces exposed to fragmentation by cosmic rays. Perhaps the most comprehensive set of semiempirical estimates of cross sections in hydrogen targets, applicable to cosmic-ray propagation, is due to Silberberg and Tsao.[3] This semiempirical formulation, applicable to targets with $Z \leq 28$, has been updated several times as new cross-section measurements have become available.[4]

Approximately, 2000–3000 partial cross sections, including their energy dependence above a few hundred MeV/nucleon, are needed for the complete cosmic-ray propagation problem. Possibly ~ 1000 of these cross sections near the peak of the mass yield distributions for each charge are actually important. The Silberberg-Tsao[3] formulation was originally based on measurements of perhaps 100 secondary reaction products, most of which were radioactive isotopes not near the peak in the mass yield distribution. The overall accuracy of this semiempirical formula for unmeasured cross sections was estimated to be $\pm 35\%$.[5] When cosmic-ray data of considerably higher accuracy became available, it became necessary to greatly improve the situation with regard to the cross sections.[5] In light of this, we embarked on a comprehensive program to measure cross sections, useful for studying cosmic-ray propagation in hydrogen and helium targets, to an accuracy of a few percent compatible with the rapidly improving cosmic-ray data set. The results of this research program have been described in papers I–III of this series.[6–8] Overall, we have now measured several hundred new cross sections, increasing the data base for understanding the cross section systematics by several fold. The 12 charges for which we have measured fragmentation cross sections comprise over 90%, by abundance, of all cosmic-ray nuclei at their source and over 70% of those arriving at earth. In addition to the isotopic and elemental cross sections, we have also measured total cross sections for a wide range of energies between 300 and 1700 MeV/nucleon. In this paper we will describe a new cross-section formula based on this data, applicable to nuclei with $Z=4-28$ and $A=7-60$ and for energies $\gtrsim 200$ MeV/nucleon, that predicts the cross sections in hydrogen targets to an accuracy $\sim 10\%$ or better.

II. THE CROSS-SECTION FORMULA

Several new systematics related to the cross sections, as described in papers I–III, have led to a great simplification over previous semiempirical formulations. These systematics will be referred to as we develop the new formulation. Because of this, we did not follow exactly any of the earlier semiempirical formulations, however, the systematics of our newer formula follow in several instances the earlier pioneering work of Rudstam.[1] In developing this formula, our procedure was to first fit all of the charge and isotropic cross sections at a single energy, 600 MeV/nucleon, where the most comprehensive measurement are available, and then to extend the formulation to other energies. This was possible because the mass fractions were observed to be essentially independent of energy.[8] In this way, the general cross-section formula can be written as a product of three essentially independent terms as follows:

$$\sigma(Z_f, A_f, E) = \sigma_0(Z_f, Z_i) f_1(Z_f, A_f, Z_i, A_i) f_2(E, Z_f, Z_i) .$$

In other words the total σ for a particular final charge Z_f, mass A_f, and energy E, depends on

(1) A term, σ_0 which is, in effect, the charge changing cross section, and depends only on the initial charge, Z_i, and final charge Z_f [see paper II (Ref. 7).

(2) A term, f_1, that defines the mean mass and half width of the mass yield distribution for a particular Z_f [see paper III (Ref. 8)]. This mean mass is a function of Z_i, A_i, and Z_f, A_f and the half width, δ_{Zf}, is a function of Z_f only [see paper III (Ref. 8)].

(3) A term f_2, which describes the overall energy dependence of the particular cross section. This term is a function of Z_i and $Z_i - Z_f$ ($=\Delta Z_f$) only [see papers I (Ref. 6) and II (Ref. 7)].

Consider now each of these terms in detail.

Term (1): This term, which describes the charge changing cross sections, is written as follows:

$$\sigma_0(Z_f, Z_i) = \sigma_{Zf} \exp\frac{-(Z_i - Z_f)}{\Delta_{Zf}} \exp\frac{-|N_{Zi} - N_{0Zf}|}{8.5} .$$

The first exponential term in this expression represents the fact that the elemental cross sections into a particular Z_f can be well represented by an exponential function of the difference $Z_i - Z_f$ for any Z_i, as discussed in paper II.[7] This fact alone permits a great simplification over earlier formulations. A similar behavior has been found for the charge changing cross sections of heavier nuclei.[9] In Table I we show the parameters σ_{Zf} and Δ_{Zf} for each Z_f between 4 and 25, as presented in paper II.[7]

The second exponential term is related to the observation that the production of a secondary nucleus with Z_f, A_f, from a primary nucleus with Z_i, A_i, is a maximum when the primary and secondary nucleus both have A_i and A_f near the mass stability line. This line of stability for β decay is shown in Fig. 17 of paper III (Ref. 8) (for convenience this figure is reproduced here as Fig. 1). This effect may be examined by studying the cross sections for a given Z_f from primaries of different neutron excess—e.g., production of $Z_f = 16$ from ^{56}Fe, ^{40}Ca, and ^{40}Ar beams. We have adopted an exponential function to describe this dependence, using the neutron excess $N = A - 2Z$, where N_0 defines the line of β stability. The characteristic width of this function is estimated from the data to be 8.5 A.

Term (2): This term, which describes the distribution of isotopic cross sections for a given element, is written as follows:

$$f_1(Z_f, A_f, Z_i, A_i) = \frac{1}{\delta_{Zf}\sqrt{2\pi}} \exp\frac{-(N - N_{Zf})^2}{2\delta_{Zf}} .$$

Here N_{Zf} is the neutron excess of the centroid of the mass distributions of the cross sections for each charge, as presented in Figs. 13–15 of paper III (Ref. 8) and summarized in Fig. 1 of this paper. Here δ_{Zf} is the characteristic width of these mass distributions as summarized in Fig. 16 of paper III.[8] The values of δ_{Zf} can be parametrized by the expression $\delta_{Zf} = 0.32 Z_f^{0.39}$ for $Z_f \geq 5$. Note that the quantity N_{Zf} must be defined separately for each Z_f in terms of Z_i, A_i, as illustrated in Fig. 1. This rather simple expression for predicting the isotopic cross section utilizes the discussion in paper III that demonstrates that to first order (1) the values of δ_{Zf} are independent of Z_i and A_i and depend only on Z_f and (2)

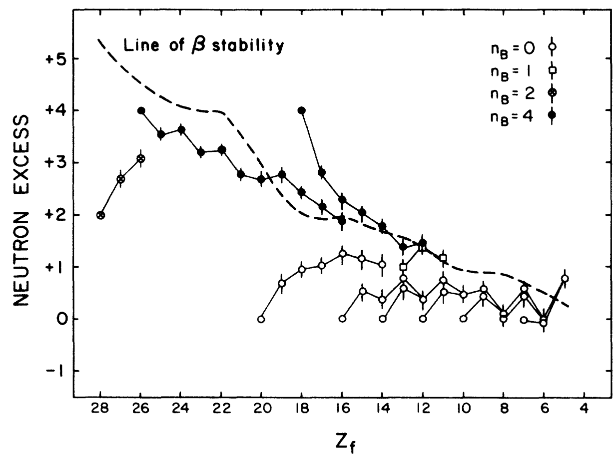

FIG. 1. Measured mean neutron excess of the elemental mass distributions as a function of fragment charge for various Z_B, A_B. Also shown is the line of β stability.

TABLE I. Fitting parameters for production of $4 \leq Z_f \leq 25$ nuclei at 600 MeV/nucleon.

Z_f	σ_{Zf} (mb)	Δ_{Zf}
25	161.4	5.7
24	154.6	8.2
23	135.7	6.2
22	158.0	6.2
21	126.6	5.9
20	160.2	5.9
19	74.8	6.9
18	144.5	4.9
17	140.1	4.5
16	142.5	5.6
15	112.5	4.9
14	145.0	6.2
13	112.0	4.3
12	134.5	6.2
11	102.5	4.1
10	99.2	5.4
9	59.2	3.1
8	99.2	6.3
7	86.6	4.8
6	94.0	6.2
5	61.2	3.9
4	19.6	6.1

these mass distributions are independent of energy thus allowing term (2) in the expression for the cross sections to be described independently of term(3), the energy dependence. We should note here that this is not strictly true for isotopes well off the stability line and also at low energies. (See also paper III.[7])

Term (3): This term, which describes the energy dependence of the cross sections, may be written separately, as per the discussions in papers II (Ref. 7) and III (Ref. 8). It is taken to be a function of Z_i and $Z_i - Z_f$ only and is written as

$$f(E,Z_i,Z_i-Z_f) = \left[1+m(\Delta Z)g(Z_i)\exp\frac{-(E-E_m)^2}{\Delta E_m}+\cdots\right].$$

The first term, illustrated here, dominates the energy range from 600–2000 MeV/nucleon. The coefficients of this term $m(\Delta Z)$ depend on the difference $Z_i - Z_f$. These coefficients and the values of E_m and ΔE_m derived from fitting the data are part of the input data file to the program. The term $g(Z_i)$ expresses the observation that the magnitude of the energy dependence is a strong function of Z_i.[7] In this case $g(Z_i)=(Z_i/26)^{2.5}$ and this term multiplies the basic form of the energy dependence given by the exponential term.

The second term in the energy dependence expression is given by

$$n(\Delta Z)h(Z_i)\exp\frac{-(E-E_n)^2}{\Delta E_n}.$$

This term is identical in form to the first term and dominates the intermediate energy range from ~ 200–600 MeV/nucleon. The coefficients $n(\Delta Z)$ and the values of E_n and ΔE_n for each $Z_i - Z_f$ are part of the input data file to the program. The term $h(Z_i)$ also expresses the fact that the magnitude of the energy dependence of the cross sections is a strong function of Z_i. In this case $h(Z_i)=(Z_i/26)^{1.4}$, indicating a less strong Z_i dependence at these lower energies.

The third term in the energy dependence is given by

$$0(\Delta Z)i(Z_i)\exp\frac{-(E-E_0)^2}{\Delta E_0}.$$

This term is identical in form to the first and second terms and dominates the low-energy range below ~ 200 MeV/nucleon. The coefficients $0(\Delta Z)$ and the values of E_0 and ΔE_0 as a function of $Z_i - Z_f$ are a part of the input data file to the program. As before, the term $i(Z_i)$ expresses the observation that the magnitude of the energy dependence itself is a strong function of Z_i. In this case $i(Z_i)=(Z_i/26)^{0.4}$.

The final term in the energy dependence is given by

$$b(\Delta Z)b(Z_i)\frac{(E-2000)}{2000},$$

where

$$2000 < E < 4000 \text{ MeV}.$$

This term modifies the dependence at energies >2.0 GeV/nucleon, where the cross sections are known to be approaching their high-energy asymptotic limit, assumed in this case to occur at 4.0 GeV/nucleon. The coefficients $b(\Delta Z)$ are a function of ΔZ only, and are negative for small ΔZ and positive for large ΔZ in accordance with the observed behavior of the cross sections. As before, the term $b(Z_i)=Z_i/26$ takes into account the fact that the magnitude of this energy dependence is a function of Z_i and is largest for ^{56}Fe interactions.

The first term σ_0, in the expression for the cross sections, is defined at 600 MeV/nucleon so the energy dependence is normalized to this energy. The coefficients for the various terms are derived from best fits to the measured energy dependences of the elemental cross sections, in particular for ^{12}C, ^{16}O, and ^{24}Mg and ^{28}Si and ^{56}Fe beams, where measurements exist at several energies. The fit to the data from these Z_i is shown in Figs. 10–15 of paper II.[7] A separate expression for the energy dependence is used for $Z_f=4$.

In addition to these three basic terms in the expression for the cross sections, there are specialized terms for neutron and proton stripping reactions as follows:

Neutron stripping: (σ in mb)

$$1n:\quad \sigma=(4.0+0.6N_{Zi})(5+0.25Z_i)\times\left[1-\frac{[Z_i-(15+2N_{Zi})]}{15}\right],$$

$$2n:\quad \sigma=1.8[1+(11.4-Z_i^{0.7})N_{Zi}],$$

$$3n:\quad \sigma=0.3[1+(7.5-Z_i^{0.5})N_{Zi}].$$

Proton Stripping: (σ in mb)

$$1p:\quad \sigma=0.50\sigma_{Zf}(0.70-0.05N_{Zi}),$$

$$2p:\quad \sigma=1.85+[2.5(Z_f+2)-23.0](1-0.23N_{Zi}),$$

and also the $1p\,1n$ reaction;

$$\sigma=\sigma_{\text{prog}}+1.6(Z_i-9)\left[1-\frac{N_i}{4}\right].$$

The measured neutron and proton stripping cross sections for hydrogen targets are given in Tables II and III of paper III.[8] In Fig. 2 of this paper we show these measurements along with the predictions for the cases of $1n$ and $1p$ reactions.

In addition to the general properties of the cross sections described by the preceding formula, including neutron and proton stripping reactions, there are several reactions that cannot be fit into this simple pattern and must be considered as special cases in the program. These include (1) cross sections into Be, including the mass distribution of fragments, (2) certain proton and neutron stripping cross sections involving ^{14}N$\rightarrow$, ^{20}Ne$\rightarrow$, and ^{58}Ni$\rightarrow$, and (3) odd-even Z_i effects for $7 < Z_i < 13$ that do not fit into the systematic pattern for both σ_{Zf}, and the mean of the mass distribution $\bar{N}_{Zf}$.

These are considered as special cases in the program. The complete details of this program, including tables

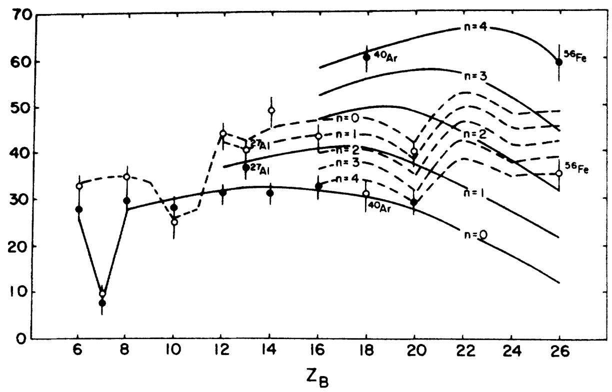

FIG. 2. Neutron and proton stripping cross sections for 1n and 1p reactions as a function of Z_B and A_B. Solid and dashed lines are predictions for 1n and 1p reactions, respectively, from the new cross section formula.

TABLE II. Comparison of measured and predicted elemental cross sections at 600 MeV/nucleon.

Beam	Number of secondary fragments	Average difference in %
^{12}C	(3)	1.8
^{14}N	(4)	3.3
^{16}O	(4)	2.8
^{20}Ne	(6)	3.8
^{24}Mg	(7)	5.6
^{27}Al	(8)	4.8
^{28}Si	(9)	4.1
^{32}S	(10)	4.9
^{40}Ar	(9)	9.8
^{40}Ca	(11)	8.5
^{56}Fe	(16)	2.9
^{58}Ni	(8)	8.2
	Σ=(95)	

and plots of the cross sections are available in limited quantities on a PC compatible diskette by writing the authors.

III. COMPARISON OF PREDICTED AND MEASURED CROSS SECTIONS

There are several ways in which the comparison between measured and predicted cross sections can be made. First considering our own set of measured cross sections, we may compare the elemental cross sections. Consider the data at ~600 MeV/nucleon. It is possible to compare the predicted and measured elemental cross sections for ~100 individual secondary fragments from the 12 beam charges we have used. In Table II we list the beam charge, the number of elemental cross sections measured, and the average percentage difference between the measurements and the predictions. The average difference is ~5% ranging from a low of 1.8% to a high of 9.8% for ^{40}Ar fragments, which is the most difficult nucleus to predict since it is well off the line of β stability. Comparisons of measurements and predictions may also be made at other fixed energies, e.g., 400 and 1000 MeV/nucleon, and show average differences ~6 to 8 %, because of the added uncertainty of the energy dependent term. This comparison between the measurements and predictions of elemental cross sections may also be seen in Figs. 10–15 of paper II.[7]

Another approach is to take our complete list of > 300 isotopic cross sections measured at ~600 MeV/nucleon as a base [Table II of paper III (Ref. 8)]. For each secondary charge for which these cross sections have been measured, we have calculated the sum of the differences between the measured and predicted cross sections for all isotopes. This sum is expressed as a fraction of the total charge changing cross section into this charge and this fraction is shown in Fig. 3, for each fragment for all of the 12 beam charges we have measured. Also shown in this figure are these average differences calculated in the same way using the Tsao and Silberberg[4] semiempirical cross section formula that has been widely used in the

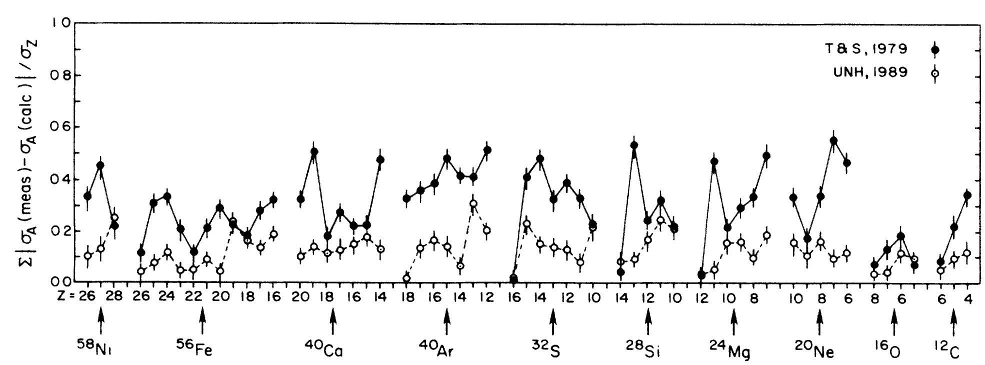

FIG. 3. Differences between measured and predicted isotopic cross sections summed for each element for secondary fragments from various beam nuclei; open circles, using formula in this paper; solid circles, using Tsao and Silberberg (Ref. 4) formula. Recent revisions of the Tsao and Silberberg formula (Silberberg *et al.*, Refs. 10 and 11) have considerably improved the fit for Fe into Ca and Ne into N and C, for example.

past. The fractional difference between the calculated and measured cross sections summed for all 12 beam charges is 10.2% for our new cross section formula and 31.6% for the Tsao and Silberberg formula. (More recent modifications to the cross section formula by Silberberg *et al.*,[10,11] have improved this fractional difference to ~26%.) These numbers are consistent with earlier estimates of a ±35% accuracy of the Tsao and Silberberg formula by Raisbeck[5] and demonstrates a significant improvement for all beam charges between our new formula and the earlier one. The average differences observed with our formula, which are ~10%, are somewhat larger than the average differences ~5% obtained from a comparison of the elemental cross sections—this difference arising from the added uncertainty of the isotopic cross-section predictions—as given by term two in the cross-section expression. One should note that the average uncertainty in the measured isotopic cross sections ranges from a few percent to >10%, so that a significant fraction of the difference between predictions and measurements could actually be due to measurement errors, rather than to errors in the cross-section formula itself.

Our prediction can also be compared with other measurements of cross sections. In a sense this has been partially done in papers II and III, where other available data has been compared with our own data base. In general, it can be seen that the agreement is excellent, e.g., in particular the extensive measurements of Fe fragmentation by Perron[12] (other individual measurements are discussed in papers II and III). We have also made this comparison with other measured cross sections in a more systematic way. Brodzinski *et al.*[13] have determined the cross sections for seven radioactive nuclides with $\sigma > 2.0$ mb from 585 MeV/nucleon proton spallation on Fe. Husain and Katcoff[14] have measured a similar number of radioactive nuclides from 3.0 GeV/nucleon proton spallation on Vanadium and Asano *et al.*[15] have made a similar measurement from 12 GeV/nucleon protons incident on Ti. These measured and the predicted cross sections are shown in Table III, along with the average difference between predictions and measurement for each experiment, determined in the same manner as was done for our cross-section data. The measurements of *V* fragmentation at 3 GeV/nucleon by Husain and Katcoff[14] show an average deviation of 20.2% between experiment and predictions. The measurements of Ti and Fe fragmentation at 12 GeV/nucleon by Asano *et al.*,[15] show average deviations of 16.9% and 29.1%, respectively. We note, however, that if the discrepancy between predictions and measurements of ^{52}Mn production from ^{56}Fe is deleted, the total average deviation for this Fe measurement becomes 19.6%. This is also true for the Fe fragmentation at 0.585 GeV/nucleon by Brodzinski,[13] which gives a total deviation = 38.1% including ^{52}Mn and 25.8% without. We emphasize this one isotope because our own measurements give 13.8 mb for this cross section at 0.6 GeV/nucleon, Perron[12] gives 14.2 mb, and Rayudu *et al.*,[16] give 14.5 mb at the same energy for the ^{52}Mn cross

TABLE III. Comparison of measured and predicted cross sections (in mb).

	Ti[a] (12 GeV/nucleon)		*V*[b] (3 GeV/nucleon)		Fe[c,a] (0.585–12 GeV/nucleon)	
	Observed	Predicted[d]	Observed	Predicted[d]	Observed	Predicted
^{54}Mn					–27.5	–26.2
^{52}Mn					6.5–4.0	16.5–12.6
^{51}Cr					29.6–21.6	32.7–21.8
^{48}V			7.5	7.9	13.7–8.0	17.9–10.7
^{47}Sc	19.8	18.9	10.7	10.1	1.9–2.0	2.1–1.7
^{46}Sc	18.4	16.1	15.2	18.3	5.4–5.0	9.7–7.2
^{44}Sc	9.6	10.7	11.7	11.1	9.3–6.7	12.8–12.5
^{43}Sc	2.1	2.3	2.6	2.8		
^{43}K	2.2	2.1	4.1	2.7		
^{42}K	6.7	7.2	7.5	7.4	2.5–2.9	3.6–3.9
Ar[e]						
^{38}Cl	3.9	9.1	3.1	6.7		
^{32}P			8.9	8.1		
^{29}Al			3.7	2.1		
^{28}Al			5.3	5.6		
^{24}Na	4.1	3.1	4.8	3.0	–3.3	–2.2
^{22}Na	2.1	2.5	2.8	2.2	–2.3	–1.5
^{21}Ne			8.7	6.6		
^{20}Ne			8.6	4.5		
	$\Sigma=68.9$	$\Delta=11.7$	$\Sigma=100.2$	$\Delta=20.3$	$\Sigma=68.9–82.7$	$\Delta+26.3–24.1$

[a]Reference 15; typical experimental errors ±6%.
[b]Reference 14; typical experimental errors ±10–15 %.
[c]Reference 13; typical experimental errors ±30%.
[d]Predictions for V and Ti are based on the individual isotropic cross sections weighted by the natural abundance of each element.
[e]Ar not shown here—discussed in paper III.

section in comparison to the observed values of 4.0–6.5 mb from Refs. 13 and 15. We thus believe that a large fraction of the deviation between predictions and measurements could be because of measurement errors themselves in all three of these comparisons.

Finally, we need to make a comment concerning the Be cross sections that are generally considered separately in our cross-section formula because their production (specifically ^{7}Be) does not fit the same pattern as heavier nuclei. Our measurements cover the production of Be from only ^{12}C, ^{14}N, and ^{16}O; for heavier nuclei production of Be we have taken the best values from an extensive survey of available cross-section data. The predicted Be cross sections should be considered to be less accurate, e.g., $\sim\pm15\%$, than the other cross sections.

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